

ordering are surrounded by atoms at a slightly greater distance than in the disordered state.

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## The Structure of $V_4Al_{23}$ \*

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The crystal structure of the intermetallic compound  $V_4Al_{23}$  has been determined. The space-group symmetry was found to be  $D_{6h}^4-P6_3/mmc$ , and precision lattice parameters were measured as  $a = 7.692_8$  Å,  $c = 17.04_0$  Å. The compound is structurally related to  $Co_2Al_5$ ,  $Fe_3NiAl_{10}$ , and  $Mn_3SiAl_6$ . The compound shows a pronounced, but imperfect, layering. Zone boundaries are prominent. Abnormally short Al-V bond distances were found.

### Introduction

A phase diagram of the aluminum-vanadium system has been published by Carlson, Kenney & Wilhelm (1955). This diagram indicates the existence of four compounds in the system, and the printed discussion of the article leaves open the possible existence of a fifth compound. In view of the interest in the compounds of aluminum with transition metals (Pratt & Raynor, 1951; Raynor & Waldron, 1948; Raynor, 1949), an examination of the structures of these aluminum-vanadium compounds has been undertaken. A determination of the structure of the compound designated as the  $\beta$ -phase, tentatively identified in the forementioned phase diagram as  $VAL_6$ , has been completed and is described in this paper.

### Sample preparation

An arc-melted aluminum-vanadium alloy containing 6.7 wt.% vanadium was heated to 710° C. and held at temperature for 175 hr. in a beryllia vessel under a helium atmosphere. At the end of the heating period the sample was quenched in an oil bath. Optical examination of the alloy indicated the presence of two phases, aluminum plus a compound. The aluminum matrix was dissolved with 1N NaOH. The residue consisted of deep metallic blue crystals in the form of long hexagonal prisms. A representative sample of this residue was selected and analyzed for vanadium (Freeland & Fritz, 1955). The analysis showed  $24.71 \pm 0.18$  wt.% vanadium. This result compares with a theoretical value of 23.94 wt.% vanadium for  $VAL_6$

and of 24.72 wt.% vanadium for  $V_4Al_{23}$ . Spectrographic analysis of the crystals showed the presence of Cu, Mn, Fe, Si, and Ti as trace impurities.

### Cell dimensions and crystal symmetry

A back-reflection Weissenberg camera, similar to the one described by Buerger (1942), was used to obtain precision lattice constants. A hexagonal unit cell was found with dimensions

$$a = 7.692_8 \text{ Å}, \quad c = 17.04_0 \text{ Å}.$$

The measured density was found to be 3.089 g.cm.<sup>-3</sup> while the density calculated on the basis of 46 aluminum atoms and 8 vanadium atoms per unit cell is 3.134 g.cm.<sup>-3</sup>.

Weissenberg patterns of the reciprocal space lattice taken about the unique axis showed  $C_{6l}$  symmetry for all observed levels,  $l = 0$  through  $l = 10$ . Thus Laue symmetry  $D_{6h}$  was indicated. No systematic extinctions were observed in  $(hkl)$  or  $(h0l)$  reflections, but  $(hhl)$  reflections occurred only with  $l = 2n$ . The Laue symmetry and extinction information limited the possible space groups to  $D_{6h}^4-P6_3/mmc$ ,  $C_{6v}^4-P6_3mc$ , or  $D_{3h}^4-P\bar{6}2c$ . Three-dimensional intensity data were taken in order to select the proper space group. These data were obtained by a multiple-film technique, using Cu  $K\alpha$  radiation and an equi-inclination Weissenberg camera. The intensities were estimated visually. The data were corrected for Lorentz and polarization factors in the manner of Lu (1943). A statistical examination of the intensity distribution was made, using the method of Howells, Phillips & Rogers (1950). The distribution curve is shown in Fig. 1 and indicates

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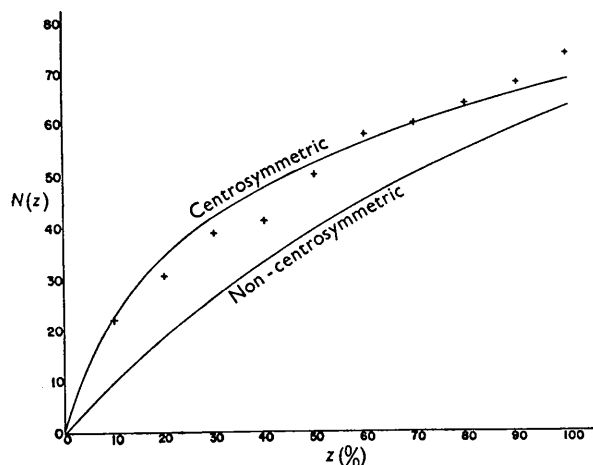


Fig. 1. Intensity distribution for the  $(hkl)$  reflections. The solid lines give the theoretical distributions.  $N(Z)$  is the number of reflections with an intensity less than or equal to  $Z\%$  of the local average intensity.

that the centrosymmetric space group,  $D_{6h}^4-P6_3/mmc$ , is the correct choice.

### Structure determination

From the intensity data the following vector projections were made:  $P(x, y)$ ,  $P(x+y, z)$ ,  $P(x, z)$ ,  $P(x, y, 0)$ ,  $P(x, y, \frac{1}{2})$ , and  $P(x, 0, z)$ . The projections indicated imperfect layering in the  $[110]$  direction. Further interpretation indicated that most of the atoms were in positions with  $x, y$  coordinates of the type  $x, 2x$  with  $x$  approximately  $7/60$ ,  $13/60$ , or  $27/60$ . Values indicated for  $z$  coordinates were of the order of  $1.5/60$ ,  $7/60$ ,  $10/60$ , and  $15/60$ . These coordinates were utilized to construct a trial structure which satisfactorily matched the vector projections. This trial structure corresponded to the formula  $V_4Al_{23}$ . Spatial considerations ruled out the possibility of utilizing another two-fold position for aluminum atoms and hence eliminated the formula  $VAI_6$ .

Additional intensity data were taken in order to refine the trial structure. The single crystal used for the recording of these data was a hexagonal prism measuring  $0.4$  mm. in length and  $0.05$  mm. in diameter. The intensities of the  $(h0l)$  and  $(hhl)$  reflections were measured visually from timed exposures taken with a precession camera, using  $Mo K\alpha$  radiation. A Geiger-Müller tube attachment to a Weissenberg camera was employed to measure the intensities of the  $(hkl)$  reflections with  $Cu K\alpha$  radiation. The intensities were put on an absolute scale, using the method described by Wilson (1942). The three-dimensional intensity data taken with  $Cu$  radiation were not used for the final structure refinement because of the difficulty in making adequate absorption correction.

Fourier projections were made on the basis of the trial structure. Good agreement was obtained between the resulting projections and the postulated structure.

The atomic parameters were refined by difference Fourier projections, using the graphical technique described by Lipson & Cochran (1953). Three successive refinements were necessary. The best values for the structure parameters are as follows:

- $2 V_1$  in  $2(a)$ :  $0, 0, 0$  parameterless;  
 $6 V_2$  in  $2(h)$ :  $x, 2x, \frac{1}{4}$  with  $x = 0.781 \pm 0.002$ ;  
 $12 Al_1$  in  $12(k)$ :  $x, 2x, z$  with  $x = 0.213 \pm 0.003$   
 and  $z = 0.029 \pm 0.001$ ;  
 $12 Al_2$  in  $12(k)$ :  $x, 2x, z$  with  $x = 0.124 \pm 0.003$   
 and  $z = 0.618 \pm 0.001$ ;  
 $12 Al_3$  in  $12(k)$ :  $x, 2x, z$  with  $x = 0.455 \pm 0.003$   
 and  $z = 0.166 \pm 0.001$ ;  
 $6 Al_4$  in  $6(h)$ :  $x, 2x, \frac{1}{4}$  with  $x = 0.126 \pm 0.002$ ;  
 $4 Al_5$  in  $4(f)$ :  $\frac{1}{3}, \frac{2}{3}, z$  with  $z = 0.167 \pm 0.001$ .

Evaluation of the standard deviations of the atomic parameters was based on the formulae of Cruickshank (1949). The final electron-density projection,  $\rho(x, z)$ ,

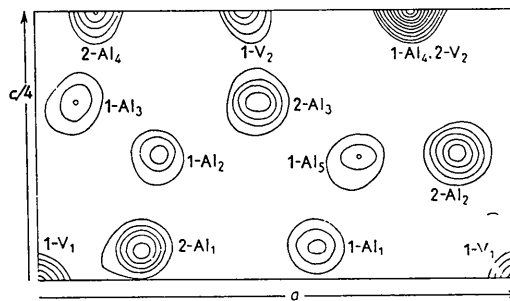


Fig. 2. The asymmetric unit of the final  $(h0l)$  electron-density projection without the use of an artificial convergence factor. The contours are drawn at intervals of 400 arbitrary units.

is shown in Fig. 2. No artificial convergence factor was used in making this projection.

Calculated structure factors based on the above parameters are compared with observed structure factors in Table 1. The reliability index

$$R = \frac{\sum ||F_o| - |F_c||}{\sum |F_o|}$$

based on the values in Table 1 is 0.14 for  $(h0l)$  data, 0.12 for  $(hhl)$  data, and 0.17 for  $(hkl)$  data. In computing these index values, the most intense low-angle

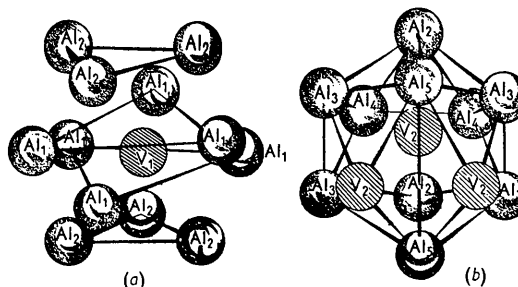


Fig. 3. The atomic arrangement (a) about the  $V_1$  atoms, (b) about the  $V_2$  atoms.

Table 1. Observed and calculated structure factors for  $V_4Al_{23}$ 

hkl	F <sub>obs</sub>	F <sub>calc</sub>	hkl	F <sub>obs</sub>	F <sub>calc</sub>	hkl	F <sub>obs</sub>	F <sub>calc</sub>	hkl	F <sub>obs</sub>	F <sub>calc</sub>
002	---	-78	302	88	-79	5,0,18	92	-90	116	20	+20
004	101	+83	303	219	+226	5,0,19	88	-101	118	41	+41
006	<12	-2	304	35	-35	5,0,20	23	+28	1,1,10	55	+52
008	242	+264	305	27	-20	6,0,18	78	-84	1,1,12	<12	-2
0,0,10	135	-121	306	33	+27	6,0,19	75	+81	1,1,14	31	+29
0,0,12	97	-82	307	58	+59	6,0,20	38	-50	1,1,16	35	+33
0,0,14	223	-246	308	30	-31	6,0,21	148	-145	1,1,18	36	+35
0,0,16	105	+82	309	<11	-3	6,0,22	24	-27	1,1,20	17	+16
0,0,18	<12	-9	3,0,10	101	+108	6,0,23	23	+14	1,1,22	25	+31
0,0,20	<12	-18	3,0,11	259	+274	6,0,24	36	+20	2,2,10	14	+12
0,0,22	177	-174	3,0,12	34	-34	6,0,25	34	+20	2,2,12	15	+9
0,0,24	162	+140	3,0,13	99	-106	6,0,26	<12	-12	2,2,14	66	+62
1,0,1	32	+25	3,0,14	115	+117	6,0,27	38	-41	2,2,16	36	+33
1,0,3	50	+41	3,0,15	37	-44	6,0,28	17	+7	2,2,18	44	+41
1,0,5	50	-43	3,0,16	<12	+14	6,0,29	72	+77	2,2,20	<11	-5
1,0,7	20	+17	3,0,17	35	-24	6,0,30	11	+13	2,2,22	18	+16
1,0,9	<12	+11	3,0,18	53	+54	6,0,31	72	+76	2,2,24	15	+9
1,0,11	24	-28	3,0,19	52	+60	6,0,32	71	+79	2,2,26	16	+12
1,0,13	163	-165	3,0,20	13	+19	6,0,33	34	-40	2,2,28	14	+13
1,0,15	87	+82	3,0,21	93	-85	6,0,34	<12	-12	2,2,30	11	-5
1,0,17	<12	+3	3,0,22	68	-81	6,0,35	32	+21	2,2,32	269	+303
1,0,19	61	+53	4,0,1	44	+39	6,0,36	38	-41	2,2,34	61	+60
1,0,21	<12	-10	4,0,2	26	+42	7,0,10	11	+13	2,2,36	15	+11
1,0,23	<11	-16	4,0,3	<11	+22	7,0,11	24	-27	2,2,38	145	+169
1,0,25	18	-26	4,0,4	103	+110	7,0,12	<11	-4	2,2,40	71	+79
1,0,27	17	+26	4,0,5	21	-35	7,0,13	<11	-7	2,2,42	70	+62
1,0,29	81	+77	4,0,6	44	+33	7,0,14	<11	-3	2,2,44	159	+166
1,0,31	69	+67	4,0,7	22	+26	7,0,15	<11	-12	2,2,46	11	-10
1,0,33	37	+46	4,0,8	<11	+7	7,0,16	10	-26	2,2,48	18	+16
1,0,35	<12	+15	4,0,9	<11	-18	7,0,17	35	-40	2,2,50	153	+160
1,0,37	<13	-9	4,0,10	24	-23	7,0,18	25	+25	2,2,52	10	-2
1,0,39	27	+24	4,0,11	24	-23	7,0,19	<11	-14	2,2,54	66	+78
1,0,41	52	+43	4,0,12	71	+63	7,0,20	31	-31	2,2,56	58	+57
1,0,43	16	+16	4,0,13	<12	-17	7,0,21	<12	-12	2,2,58	22	+16
1,0,45	54	+47	4,0,14	<12	+3	7,0,22	<11	-20	2,2,60	34	-31
1,0,47	<12	+15	4,0,15	<12	-6	7,0,23	18	-26	2,2,62	37	+41
1,0,49	14	+13	4,0,16	<12	-6	7,0,24	10	-26	2,2,64	61	+59
1,0,51	21	+13	4,0,17	19	-24	7,0,25	<11	-1	2,2,66	34	-31
1,0,53	15	+13	4,0,18	18	-30	7,0,26	37	+33	2,2,68	37	+41
1,0,55	68	+63	4,0,19	25	+33	7,0,27	24	+20	2,2,70	133	+127
1,0,57	176	+151	4,0,20	30	+31	8,0,1	<11	+1	2,2,72	230	+211
1,0,59	123	-91	4,0,21	505	-35	8,0,2	<11	+1	2,2,74	12	+5
1,0,61	130	-101	4,0,22	85	-76	8,0,3	34	+15	2,2,76	69	+61
1,0,63	30	-36	4,0,23	72	+71	8,0,4	31	-31	2,2,78	11	-10
1,0,65	36	+36	4,0,24	111	+109	8,0,5	34	-39	2,2,80	51	+47
1,0,67	37	-26	4,0,25	161	+169	8,0,6	34	-39	2,2,82	8	-8
1,0,69	106	+93	4,0,26	144	+132	8,0,7	107	+122	2,2,84	29	+29
1,0,71	39	-28	4,0,27	<11	-6	8,0,8	65	+72	2,2,86	55	+52
1,0,73	101	+93	4,0,28	<11	-13	8,0,9	<11	-4	2,2,88	31	+29
1,0,75	60	+49	4,0,29	27	+26	8,0,10	14	-10	2,2,90	31	+29
1,0,77	38	-39	4,0,30	<11	+1	8,0,11	14	-10	2,2,92	31	+29
1,0,79	<13	+5	4,0,31	28	-23	8,0,12	13	-19	2,2,94	31	+29
1,0,81	53	+46	4,0,32	59	-56	8,0,13	125	+176	2,2,96	12	+5
1,0,83	27	+26	4,0,33	18	-21	8,0,14	<11	-3	2,2,98	69	+61
1,0,85	<13	+7	4,0,34	<12	+1	8,0,15	<11	-8	2,2,100	11	-10
1,0,87	35	+27	4,0,35	<12	-13	8,0,16	12	+10	2,2,102	8	-8
1,0,89	63	+55	4,0,36	<12	-19	8,0,17	20	+25	2,2,104	29	+29
1,0,91	179	-154	4,0,37	<12	-19	8,0,18	50	+60	2,2,106	55	+52
1,0,93	121	-124	4,0,38	59	+60	8,0,19	<12	-1	2,2,108	72	+68

Table 2. Interatomic distances in  $V_4Al_{23}$ 

Atom	Neighbor	No. of neighbors	Distance (Å)	Probable error (Å)
$V_1$	$Al_1$	6	2.88	0.020
	$Al_2$	6	2.60	0.025
$V_2$	$V_2$	2	2.64	0.020
	$Al_2$	2	2.58	0.035
	$Al_3$	4	2.83	0.035
	$Al_4$	2	2.54	0.025
	$Al_5$	2	2.73	0.030
$Al_1$	$V_1$	1	2.88	0.020
	$Al_1$	2	2.78	0.025
	$Al_1$	2	3.00	0.030
	$Al_2$	2	2.77	0.035
	$Al_3$	2	2.90	0.035
$Al_2$	$Al_3$	2	2.84	0.040
	$Al_5$	1	2.96	0.030
$Al_3$	$V_1$	1	2.60	0.025
	$V_2$	1	2.58	0.035
	$Al_1$	1	2.77	0.030
	$Al_2$	2	2.90	0.030
	$Al_3$	2	2.86	0.030
$Al_4$	$Al_2$	2	2.92	0.035
	$Al_3$	2	2.80	0.030
	$Al_5$	1	2.79	0.025
$Al_5$	$V_2$	2	2.83	0.035
	$Al_1$	2	2.84	0.040
	$Al_2$	2	2.92	0.035
	$Al_3$	2	2.81	0.035
	$Al_4$	1	2.86	0.030
$Al_5$	$Al_4$	2	2.80	0.030
	$Al_5$	1	2.95	0.025
$Al_4$	$V_2$	2	2.54	0.025
	$Al_2$	4	2.80	0.030
	$Al_3$	4	2.80	0.030
	$Al_4$	2	2.91	0.020
$Al_5$	$V_2$	3	2.73	0.030
	$Al_1$	3	2.96	0.030
	$Al_2$	3	2.79	0.025
	$Al_3$	3	2.95	0.025

reflection, (330), is omitted. Temperature factors of  $0.34 \text{ \AA}^2$  for the ( $h0l$ ) data and  $0.50 \text{ \AA}^2$  for the ( $hhl$ ) data were found to give the best agreement between observed and calculated structure factors. The ( $hkl$ ) data were not corrected for the temperature factor.

Fig. 3 shows the atomic arrangements about the two types of vanadium positions,  $V_1$  and  $V_2$ , respectively. In Fig. 3(a) the  $V_1$  atom is situated at the origin. The connected aluminum atoms lie in planes normal to the  $c$  axis and are related by a  $6_3$  screw axis. In Fig. 3(b) the  $V_2$  atoms are related by a threefold rotation axis in the  $[001]$  direction passing through the  $Al_5$  atoms. The  $V_2$  and  $Al_4$  atoms lie on a plane normal to and one-quarter along the  $c$  axis. An interesting feature of the clustering around the two types of vanadium atoms is that in both cases the surrounding atoms approximate regular icosahedra. Pauling (1947) has pointed out that the most efficient C.N. 12 packing is represented by a regular icosahedron if the radius of the central sphere is 0.901 times the radius of spheres at the vertices of the icosahedron. The radius ratio of vanadium to aluminum is 0.923.

Each atom in the structure has twelve neighbors within  $3.0 \text{ \AA}$ . Table 2 lists the interatomic distances. The average values for the Al-Al and V-V bonds in the structure are the same as in the pure metals,  $2.86 \text{ \AA}$  and  $2.64 \text{ \AA}$ , respectively. The mean Al-V bond distance is  $2.72 \text{ \AA}$ , while that predicted on the basis of the interatomic distances in the pure metals

is  $2.77 \text{ \AA}$ . The  $2.77 \text{ \AA}$  distance is computed for C.N. 12 using the formula of Pauling (1947). The volume occupied by the Al and V atoms in  $V_4Al_{23}$  is 0.3% less than the volume these atoms would occupy in the pure metals. The shortest observed bond distance is  $2.54 \text{ \AA}$  for the  $Al_4$ - $V_2$  bond. The only other compound in the aluminum-vanadium system whose structure has been determined is  $VAL_3$  (Brauer, 1943). In  $VAL_3$  the average Al-V bond is  $2.74 \text{ \AA}$  and the shortest Al-V bond is  $2.67 \text{ \AA}$ . Thus the aluminum-rich  $V_4Al_{23}$  has some significantly shorter Al-V bonds than those occurring in  $VAL_3$ .

### Discussion

The structure of  $V_4Al_{23}$  is closely related to the structures of  $Co_2Al_5$  (Bradley & Cheng, 1938),  $Fe_3NiAl_{10}$  (Bradley & Taylor, 1940), and  $Mn_3SiAl_9$  (Robinson, 1952). Robinson has examined the geometry of the Brillouin zone for these structures and has found that

an inscribed prolate spheroid in the zone should fill at an electron/atom ratio near 1.68. It has been pointed out by Robinson and also by Taylor (1954) that the most stable composition for some of the aluminum-transition-element compounds is non-stoichiometric and occurs with a deficit of transition element atoms. Apparently no such deficit occurs in  $V_4Al_{23}$ . In the phase diagram  $V_4Al_{23}$  is shown as a line compound with negligible composition variation, and the fore-mentioned chemical analysis indicates that this composition is stoichiometric. Table 3 shows the computed electron/atom ratios for  $V_4Al_{23}$  and the related compounds at stoichiometry and at the estimated stable composition.

Table 3. *Electron/atom ratios for  $V_4Al_{23}$  and related compounds\**

Compound	Electron/atom ratio	
	(Stoichiometric)	(Non-stoichiometric)
$V_4Al_{23}$	1.72	—
$Mn_3SiAl_9$	1.54	1.78 (19.60 at. % Mn, 8.88 at. % Si; Robinson) 1.59 (22.05 at. % Mn, 6.50 at. % Si; Taylor)
$Co_2Al_5$	1.65	1.72 (27.2 at. %; Robinson, Taylor)
$Fe_3NiAl_{10}$	1.53	1.68 (17.7 at. % Fe, 8.7 at. % Ni; Robinson)

\* Computed with the following values for valence:

Al +3	V —5.66	Fe —2.66	Ni —0.61
Si +4	Mn —3.66	Co —1.71	

The consistency of the electron/atom ratios at stable composition lends support to arguments based on zone theory. Of the four structures in question only  $Co_2Al_5$  and  $Fe_3NiAl_{10}$  are isomorphous. However, all four of the structures have the same space group symmetry which gives rise to identical Brillouin zone geometry.

The transition metal configuration in the above four structures has one feature in common: three transition metal atoms are bonded as nearest neighbors in an equilateral triangle. This differs from most of the aluminum-transition-metal structures in that the transition metal atoms are not usually nearest neighbors. Other common characteristics of aluminum-transition-element structures are exhibited by these four compounds. These include layering, short aluminum-transition-metal bond distances, and prominent zone boundaries. In  $V_4Al_{23}$  the layering occurs normal to the [110] and [120] directions.

Black (1956) has discussed the structural relationships between the compounds of aluminum with transition elements. He has noted a trend of an increase in coordination to aluminum with decreasing atomic number of the transition element: from eight aluminum atoms coordinated to copper to ten aluminums coordinated to manganese. The coordination in  $V_4Al_{23}$  is consistent with this trend; 25% of the vanadium atoms are coordinated to twelve aluminums and 75%

of the vanadium atoms are coordinated to ten aluminums.

Measurements of magnetic susceptibility by Föex & Wucher (1954) indicate that in aluminum-transition-element compounds pairing of the  $d$  electrons occurs. On this basis  $V_4Al_{23}$  would be expected to be weakly paramagnetic with little temperature dependence. The filling of non-bonding sub-shells and the contraction of bond distances seem somewhat contradictory, and hence the evidence supports the view that the  $d$  electrons are involved in the bonding. This is not inconsistent with the susceptibility measurements of Föex & Wucher since, as a first approximation (Wilson, 1953), temperature-independent paramagnetism is a function only of the Fermi level. The introduction of transition metal atoms into aluminum with the high density of states in the  $d$  levels would tend to lower the Fermi level. Hence, a susceptibility less than pure aluminum observed in  $CrAl_3$  and  $MnAl_6$  is not unreasonable and might also be expected in  $V_4Al_{23}$ .

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